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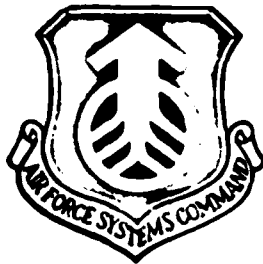UNCLASSIFIED   FTD-ID(RS)T-0960-81   NL

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	ъ ъ	*ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	ь ь	*ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$sinh^{-1}$
cos	cos	ch	cosh	arc ch	$cosh^{-1}$
tg	tan	th	tanh	arc th	$tann^{-1}$
ctg	cot	cth	coth	arc cth	$coth^{-1}$
sec	sec	sch	sech	arc sch	$sech^{-1}$
cosec	csc	csch	csch	arc csch	$csch^{-1}$

Russian	English
rot	curl
lg	log

THE FORMATION OF RADIATION PATTERNS OF ARC ANTENNAS

V.M. Golovachev, A.A. Kuz'min

The formula is obtained for calculation of the radiation pattern of arc antennas in a plane orthogonal to the plane of an arc. It is shown that the formula of the radiation pattern depends on the radius of the arc and amplitude distribution in the orthogonal plane. Characteristics of the forming of the radiation pattern of an arc antenna in a plane normal to the plane of an arc are examined.

Introduction

In the last decade an enormous number of works, [1], [2], [3], [4], and others, has been devoted to the investigation of characteristics of arc antenna arrays. This interest is explained by the definite advantages of circular antennas over plane antennas, the main thing consisting in the possibility of a wide-angle movement of the radiation pattern without a change in its characteristics. However, in the overwhelming majority of the works published, an examination is made of characteristics of circular (arc) antenna arrays only in the plane of the arc (azimuthal plane), although it is known about the interdependence of the radiation pattern in the plane of the arc and orthogonal to it. This interdependence appears in the fact that the arc (circular) antenna array possesses directional properties in a vertical plane, even with the use of separate radiators nondirectional in this plane. The indicated directional

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properties are determined by the vertical radiation pattern corres-
ponding to factor of the arc (circular) antenna. Therefore, with the
formation in the vertical plane of special radiation patterns (of the
type $cosec^2 \theta$, sector, $cos^2 \theta$ and others), it is necessary to con-
sider the radiation pattern corresponding to the factor of the arc.
The latter, as is known [2], is determined by the geometric dimen-
sions of the arc (radius R, magnitude of aperture 2θ), the radiation
pattern of the radiators and the amplitude distribution in the azi-
muthal plane. However, the formula obtained in work [2] for calcu-
lation of the vertical radiation pattern in practice is correct only
for the calculation of the continuous arc antennas. Examined below
are problems of the formation of the radiation pattern in a vertical
plane of arc antennas with discrete distribution of the radiators.

Radiation pattern corresponding to the factor of the arc

The radiation pattern according to the field by an arc symmetric
with respect to the center of the antenna, which consists of N radi-
ators having directivity in the plane of the arc $F_u(\varphi)$ and with
an amplitude distribution $\{I_j\}$ is determined by the formula

$$F(\varphi, \theta) = \sum_{j=-n}^{n} I_j F_u(\varphi - \gamma_j)\, e^{-i[\xi_j(\varphi, \theta) - \xi_j(\varphi_0, \theta_0)]}, \tag{1}$$

where α is the angular distance between the radiators, and $\xi_j(\varphi,\theta)$
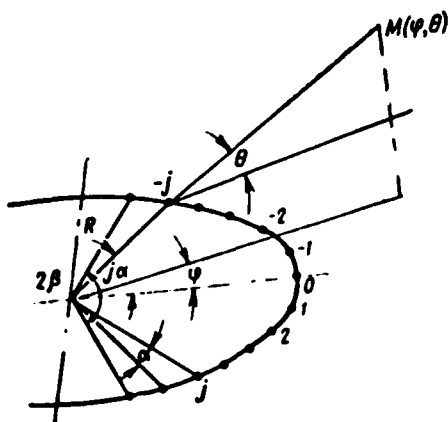is the phase factor of the arc antenna.

Fig. 1.

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The expression for the phase factor $\xi_j(\varphi, \theta)$ of the arc antenna, which can easily be obtained from Fig. 1, has the form

$$\xi_j(\varphi, \theta) = \kappa R \cos\theta \left[(1 - \cos\gamma_j)\cos\varphi + \sin\gamma_j \sin\varphi \right]. \tag{2a}$$

To ensure the cophasal addition of fields of all the radiators in the assigned direction φ_0, θ_0, the electrical phase compensation for the j-th radiator must be equal to

$$\xi_j(\varphi_0, \theta_0) = \kappa R \cos\theta_0 \left[(1 - \cos\gamma_j)\cos\varphi_0 + \sin\gamma_j \sin\varphi_0 \right]. \tag{2b}$$

where φ_0, θ_0 are the angles determining the direction of the maximum of the radiaton pattern of the arc antenna, and R is the radius of the arc.

Consequently,

$$\xi_j(\varphi, \theta) - \xi_j(\varphi_0\theta_0) = \kappa R \left\{ (1 - \cos\gamma_j)(\cos\varphi\cos\theta - \cos\varphi_0\cos\theta_0) + \right.$$
$$\left. + \sin\gamma_j(\sin\varphi\cos\theta - \sin\varphi_0\cos\theta_0) \right\}. \tag{3}$$

However, for the majority of the practically realizable cases, it is required to ensure the movement of the main beam only in an azimuthal plane (plane of the arc), and therefore, it is expedient to assume that $\theta_0 = 0$.

To simplify the analysis of the radiation pattern in the plane normal to the plane of the arc (in a vertical plane), which is of interest at a given moment, let us assume that the maximum of the main beam in an orthogonal plane has the direction $\varphi_0 = 0$. Then the radiation pattern in the vertical plane corresponding to the factor of the arc is written as

$$F(\theta) = \sum_{j=-n}^{n} I_j F_n(\gamma_j) \exp\left\{ i\kappa R(1 - \cos\gamma_j)(1 - \cos\theta) \right\}. \tag{4}$$

The factor $F_n(\alpha_j)$ in expression (4) defines the contribution in the field of radiation of the arc owing to the directed properties of the radiators in the azimuthal plane in the direction $\varphi = 0$. It is obvious that this factor will be determined by the function describing the radiation pattern of the radiator in the azimuthal plane and will fulfill the role of the importance to the amplitude distribution. Let us rewrite the formula (4) in the form of

$$F(u) = e^{-iu} \sum_{j=-n}^{n} I_j' \exp\left[iu \cos\gamma_j \right]. \tag{5}$$

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where $u = kR(1 - \cos\theta)$ is the generalized coordinate, and

$\{I'_j\} = \{I_j\} F_u(\alpha_i)$ is the normalized amplitude distribution in the plane of the arc. Then, by using the equality

$$\exp(i\,u\cos\alpha_i) = \sum_{l=-\infty}^{\infty} i^l J_l(u)\exp(i\,l\,\alpha_i), \qquad \text{we get}$$

$$F(u) = \sum_{j=-n}^{n} I'_j \sum_{l=-\infty}^{\infty} i^l J_l(u)\exp(i\,l\,\alpha_i). \tag{6}$$

From expression (6) it follows that the vertical radiation pattern fo the arc antenna, in the same way as the horizontal, has a complex character and generally has no nulls.

If the radiators of the arc antenna are isotropic and uniformly excited, i.e., $\{I'_j\}$ = const, then the expression of the vertical radiation pattern acquires the form

$$F(u) = \sum_{l=-\infty}^{\infty} i^l J_l(u) \sum_{j=-n}^{n} \exp(i\,l\,\alpha_j). \tag{7}$$

By noting that $\alpha_j = \dfrac{2\beta}{N} j$ and $\displaystyle\sum_{j=-n}^{n} \exp(i\,l\,\alpha_j) = \dfrac{\sin l\beta}{\sin l\frac{\beta}{N}}$, we finally find

$$F(u) = J_0(u) + \sum_{l'=-\infty}^{\infty} i^{l'} J_{l'}(u)\,\frac{\sin l'\beta}{N\sin l'\frac{\beta}{N}}. \tag{8}$$

The prime at the subscript of summation l denotes the absence under the sum sign of the value with $l = 0$.

From (8), for the complete ring of the radiators, it is easy to obtain the expression for the vertical radiation pattern of the form

$$F(u) = J_0(u) + \sum_{m'=-\infty}^{\infty} i^{m'N} J_{m'N}(u). \tag{9}$$

If the circular [ring] array with a sufficiently large number of radiators N such that N > u is used, then the vertical radiation pattern of such an array is written as

$$F(u) \simeq J_0(u) = J_0[\kappa R(1 - \cos\theta)]. \tag{10}$$

Expression (10) completely concurs with the expression obtained in work [3] for the continuous circular aperture. Furthermore, from (8) it is easy to show that when $\beta \to 0$ the radiation pattern in the vertical plane becomes isotropic, which corresponds to the case of the line-source antenna. For circular antennas with the magnitude of the radating sector $2\beta \leqslant \pi$, for which expression (10) is incorrect,

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it is necessary to use formula (8). At a sufficiently large number
of radiators N ($N \rightarrow \infty$), expression (8) is still simplified and takes
the form

$$F(u) = J_0(u) + \sum_{l'=-\infty}^{\infty} i^{l'} J_{l'}(u) \frac{\sin l'\beta}{l'\beta} .$$
(11)

If we are limited by the first several terms of expression (11), then
for the calculation of the radiation pattern of the arc antennas in
the vertical plane, we can use the relation

$$2\Delta\theta_{0,5}^{\circ} = \frac{173^{\circ}}{\sin\frac{\beta}{2} \sqrt{\kappa R}} .$$
(12)

The accuracy of the calculation according to formula (12) is no
worse than ±5%. From equation (12) it is evident that the width of
the radiation pattern is determined by radius R and the magnitude of
the radiating sector 2β of the arc antenna. Figure 2 gives the cal-
culation dependences of the width of the radiation pattern of the arc
antennas on the magnitude of the angular aperture 2β for different
values of kR. From the given curves it is evident that with an in-
crease in values of kR, there occurs a considerable increase in the
width of the radiation pattern, especially for the small angular
apertures 2β of the arc antennas. From a physical point of view,
this is explained by the decrease in the magnitude of the phase leads
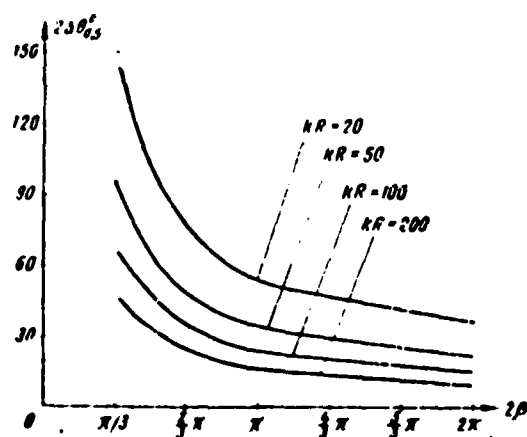caused by the geometry of the arc antennas.

Fig. 2.

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Results of the calculation

In order to investigate the change in the shape .of the vertical radiation pattern of the arc antennas at different amplitude distributions in the plane of the arc according to formulas (1) and (6), a number of numerical calculations was conducted. The calculations according to formula (1) was produced on the computer Ural-2, and those according to formula (6), manually. From the diagrams given below (see Fig. 3), we see the good concurrence of the points calculated on the digital computer and manually (the calculation values according to formula (6) are noted on the diagrams by dots).

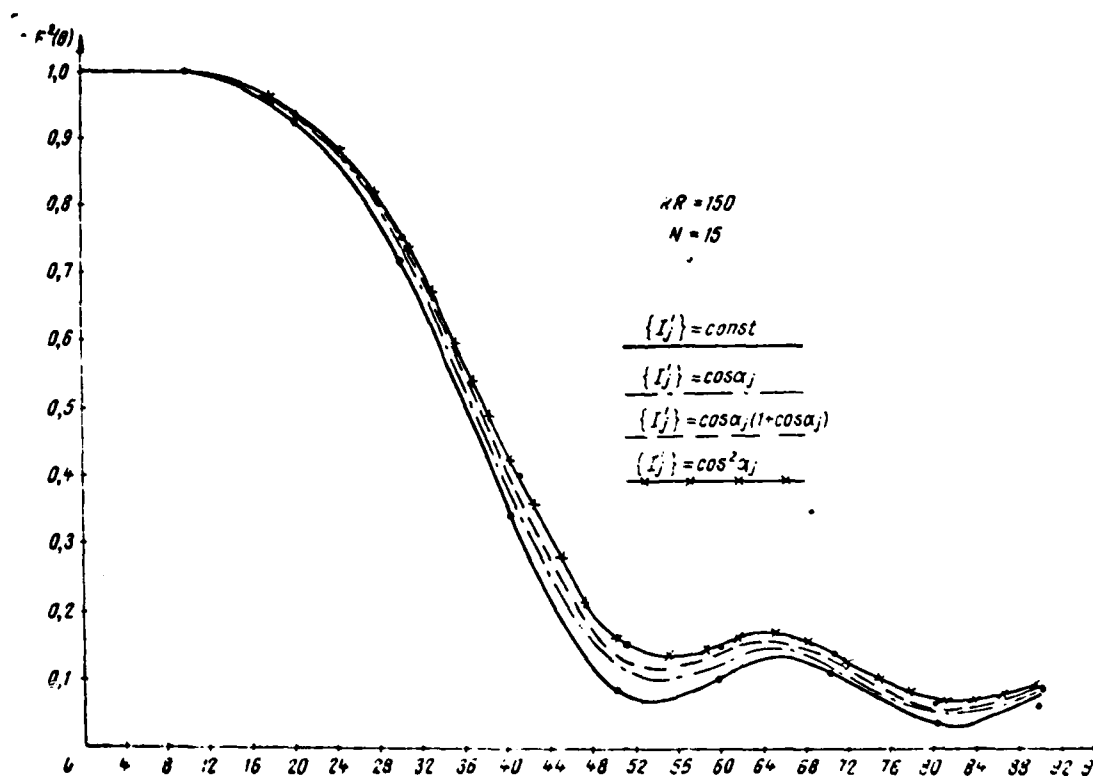Fig. 3.

Figures 3-8 give calculation radiation patterns illustrating the effect of the different amplitude distributions for the arc antennas with a curvature determined by values of kR = 50 and kR = 150 with angular apertures of 2β = 90°, 135° and 180°.

From an examination of the presented figures, it is evident that the shape of the vertical radiation pattern, which corresponds to the

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factor of the arc antennas, is close to being column-shaped. This is
explained by the fact that at small angles of elevation θ, the out-
-of-phase, caused by the change in the difference of the course of
the beams from different sections of the arc with respect to the
plane $\theta = 0$, is insignificant, and the radiation field is almost
equal to the maximum value. With a further increase in the angles
θ, out-of-phase begins to be affected, and the field in the far
zone sharply decreases. The presence of a nonuniform amplitude dis-
tribution in the plane of the arc leads to an expansion of the radia-
tion pattern in the orthogonal plane owing to an increase in the
contribution to the total field of radiation from elements located
on edges of the aperture. Besides this, as was indicated above, the
radiation patterns do not have nulls, and with an increase in the
nonuniformity of the amplitude distribution, we observed the simplest
transition from the oscillatory process of the change in the signal
beyond limits of the main beam to the smooth "pulling-in" of its
slope. At relatively small angular apertures $(2\beta \leqslant 90^{\circ})$ of the arc
antenna, a certain stationarity of the shape of the radiation pat-
tern with respect to the form of the amplitude distribution is ob-
served. This occurs due to a lowering of the nonuniformity of the
amplitude distribution within the small angles 2β of aperture of the
arc antenna and of the approach of it to the line-source antenna, in
which the radiation pattern in the vertical plane generally does not
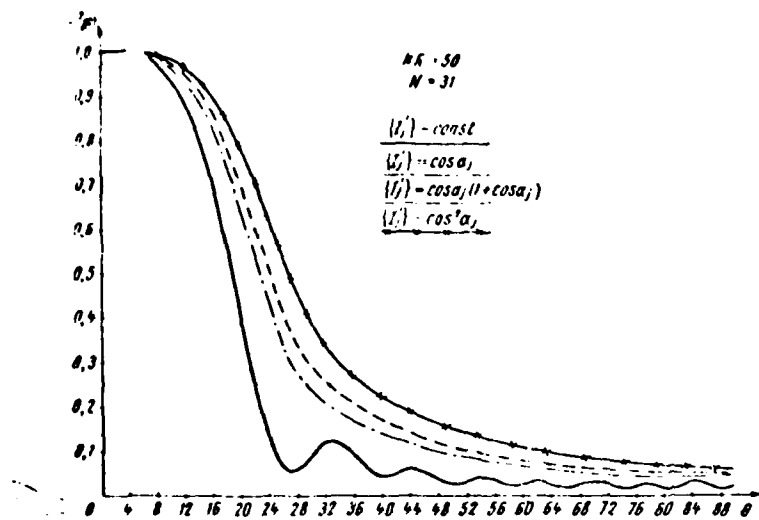depend on the amplitude distribution in the azimuthal plane.

Fig. 4.

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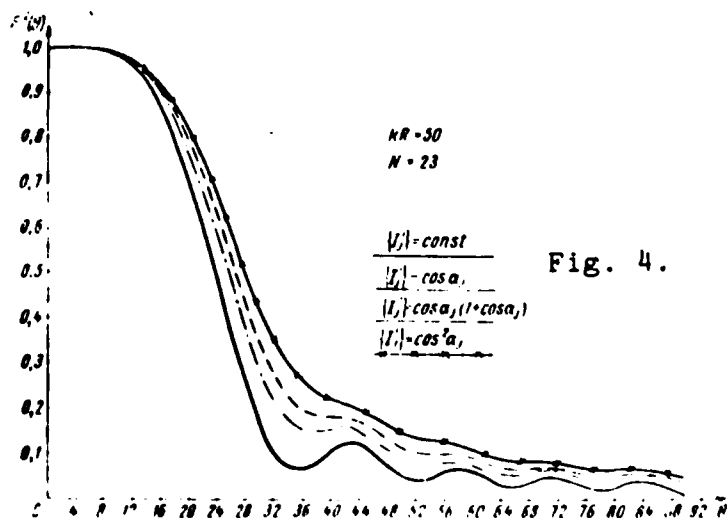
Fig. 5.
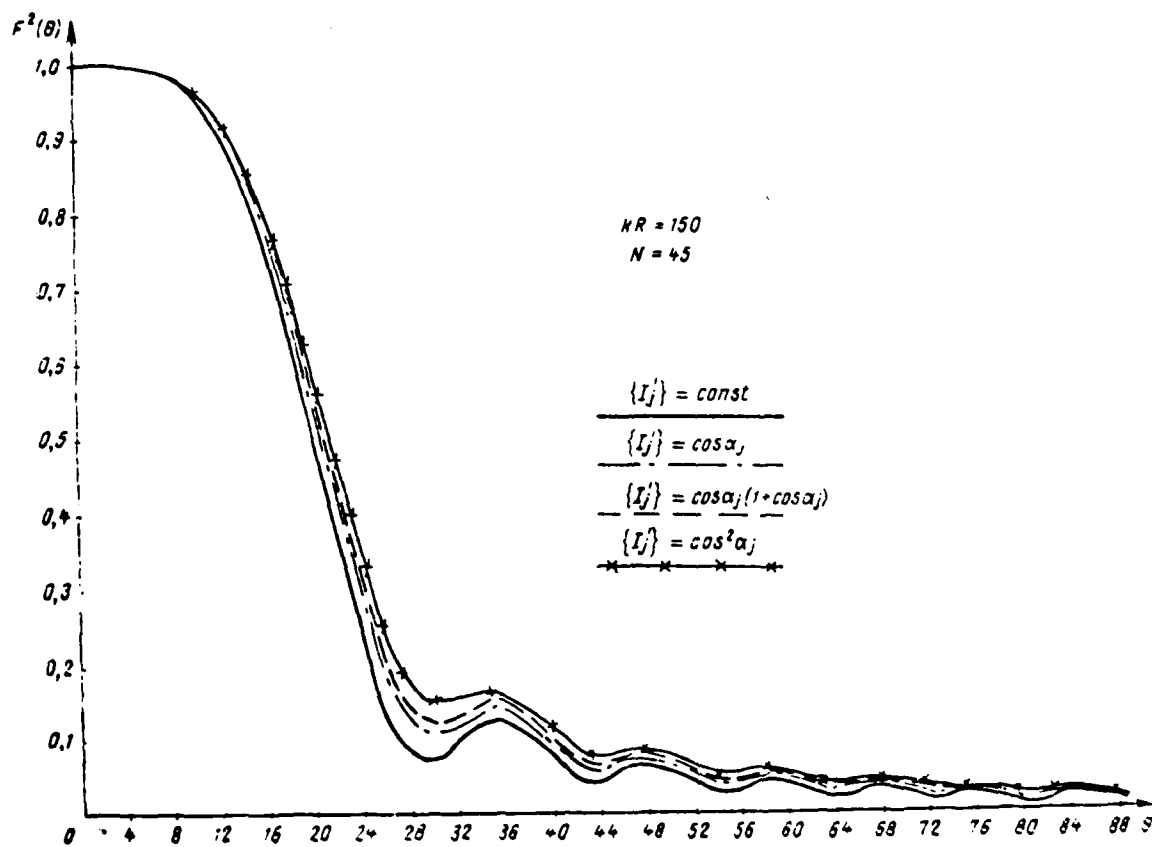
Fig. 6.
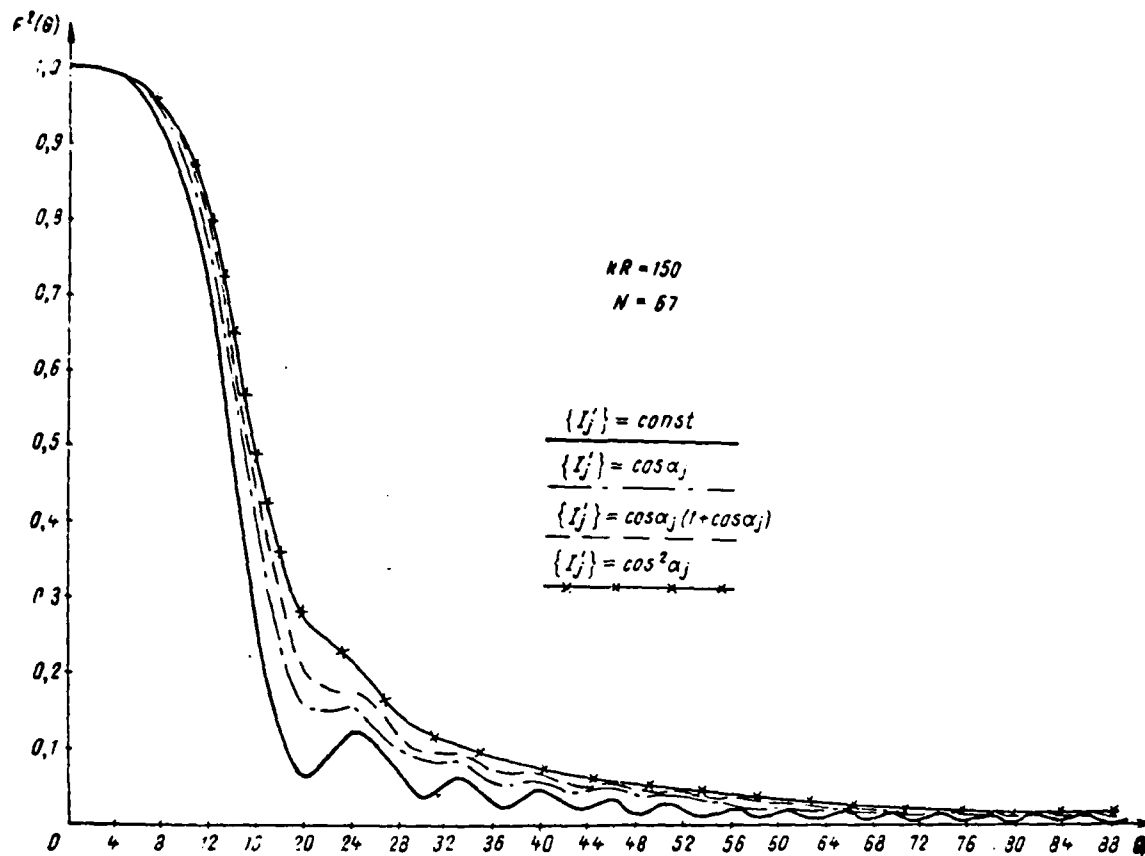
Fig. 7.

Thus from the preceding section and the analysis of the calcula-
tion radiation patterns corresponding to the factor   of the arc,
it follows that the arc array, which consists of radiators nondi-
rectional in a vertical plane, possesses directional properties.
The latter fact imposes definite requirements on the formation of
the radiation patterns of the arc arrays required in a vertical plane,
especially in the formation of the wide-directional patterns or pat-
terns of a special shape ($cosec^2 \theta$, sector and others).

Characteristics of the forming of the assigned radiation pattern in
the vertical plane in arc antennas

    In the designing of arc antenna arrays, there usually are assigned
radiation patterns both in the plane of the arc and orthogonal to it.
According to the assigned characteristics of the radiation pattern in
the plane of the arc - the width, level of side lobes, scanning

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sector, and others, the geometric dimensions of the arc (radius R, aperture 2β) and also the necessary normalized amplitude distribution $\{I'_j\}$ are determined.
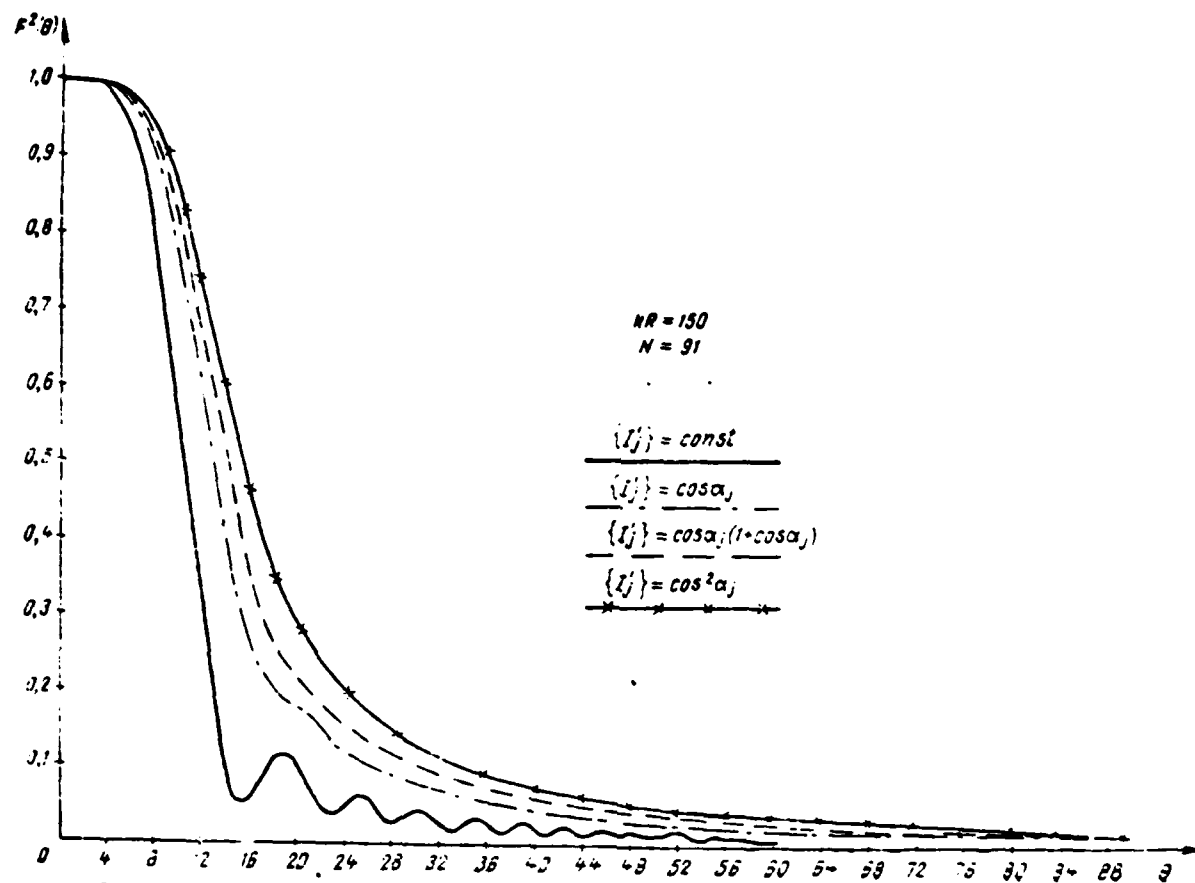
Fig. 8.

These parameters are necessary for the calculation of the verti-
cal radiation pattern corresponding to the factor of the arc antenna
array. For this the works [1] and [2] can be used. Let us emphasize
once more that in the preceding sections we considered the vertical
radiation patterns corresponding to the factor of the arc antenna
array, which consists of radiators nondirectional in a vertical plane.
The radiators usually used in the antennas possess a definite direc-
tivity in both planes. For the majority of the practical cases,
with a sufficient degree of accuracy it is possible to consider that
the three-dimensional pattern of the radiator is represented in the
form

$$F_{изл}(\psi, \theta) = f(\psi) F_1(\theta),$$

(13)

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where $f(\varphi)$ is the radiation pattern of the radiator in the plane of the arc; $F_1(\theta)$ - radiation pattern of the radiator in the plane normal to the plane of the arc; φ, θ - angles determined by directional cosines.
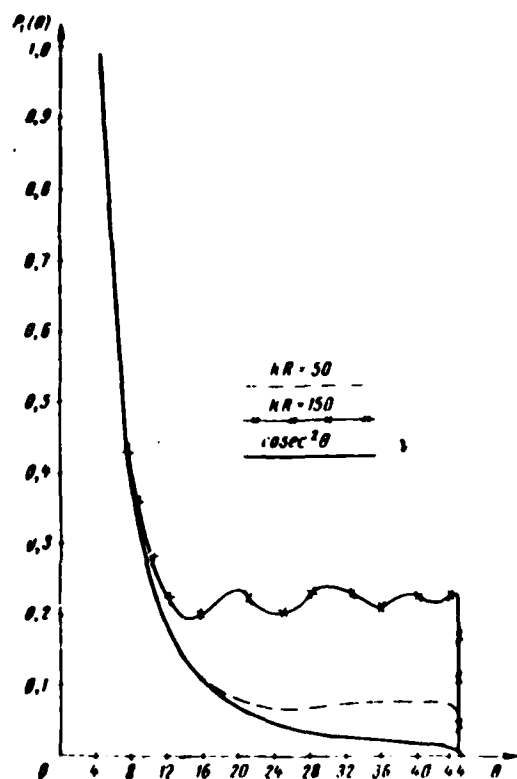
Fig. 9.

Considering condition (13), we can assume that the vertical radiation patterns of the separate radiators in different cross sections φ_n are identical, i.e.,

$$F_1(\theta) \quad F_2(\theta) \quad \ldots \quad F_n(\theta),$$

and the complete (required) radiation patterna in the plane under consideration is written in the form

$$G(\theta) \quad F(\theta) F_1(\theta), \tag{13a}$$

where $G(\theta)$ is the required radiation pattern in the vertical plane; $F_1(\theta)$ - the radiation pattern in the vertical plane of a separate radiator; $F(\theta)$ - the vertical radiation pattern corresponding to the factor of the arc array calculated according to formula (6). From

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expression (13a) it follows that for the creation of the required
vertical radiation pattern in the arc antenna, it is necessary that
the radiation pattern of the separate radiator satisfy the condition

$$F_1(\theta) \quad \frac{G(\alpha)}{I(\alpha)} \tag{14}$$

It is obvious that the form of
It is obvious that$_\wedge$the radiation pattern $F_1(\theta)$ of the separate radia-
tor will be determined both by the geometric dimensions of the arc
and the shape of the required radiation pattern.

For clarity, Fig. 9 gives the change in the shape of the verti-
cal radiation pattern with respect to power $P_1(\theta)$ of the separate
radiator installed into an arc antenna with geometric dimensions
$2\beta = 135^\circ$; kR = 50; 150 and $\{I'_j\} = \cos^2\alpha_j$ for the forming of the
radiation pattern, which is changed in the sector of $5^\circ \leq 0 \leq 45^\circ$
according to the law of $\csc^2\theta$. On this same Fig. 9, given for a
comparison is an ideal radiation pattern (solid line) of the type
$\csc^2\theta$, which should be formed by the radiator located in the line-
-source antenna array. The results given on Fig. 9 confirm that said
above, namely, that with the formation of the wide-directional radia-
tion patterns in the arc antennas there can occur a considerable
change in the shape of the appropriate radiation pattern of the sep-
arate radiator. Further, the problem is reduced to the synthesis of
a separate radiator according to the assigned radiation pattern de-
termined by formula (14).

Conclusions

1. The arc antenna array, which consists of isotropic radiators,
possesses directional properties both in the plane of the arc and in
the plane orthogonal to it. The obtained relation shows that the
width of the radiation pattern of the arc antenna in a plane normal
to the plane of the arc considerably depends on the geometric dimen-
sions of the arc.

2. The effect of the different amplitude distributions on the
radiation pattern of the arc antenna array in the plane normal to the
plane of the arc, as the numerical calculations showed, becomes
noticeable with an angular aperture of the antenna of $2\beta > 90^\circ$.

3. The radiation patterns of the whole antenna and separate rad-
iator in the plane normal to the plane of the arc are considerably

different, which should be considered with the designing of arc antennas with a radiation pattern of special shape in the plane indicated above.

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ENERGY CHARACTERISTICS OF PERIODIC SYSTEMS OF RADIATORS
Efficiency and Output Coefficient of Power, Directive Gain, and
Gain Factor

I.V. Guzeyev, A.B. Kolot

Obtained in this article, which is a continuation of [1],
are expressions for the efficiency and output coefficient; and
the connection between the matrix of scattering and partial gain
of the array element is established.

For the linear periodic array with several active radiators, the
feeders of which are excited by external incident waves, the effic-
iency equals

$$\eta = \frac{W_{изл}}{W_{пад}} = \frac{\sum\limits_{n=-\infty}^{\infty} \{|a_n|^2 - |b_n|^2\}}{\sum\limits_{p=p_1, \; p_2, \ldots, \; p_\alpha} |a_p|^2} . \tag{1}$$

where p_1, p_2, ..., p_α are numbers of the active elements; $a_n b_n$ - the
complex amplitudes of the direct and inverse waves in feeders of the
elements.

On the basis of [1], which was used by the Parseval theorem,
relation (1) can be given the form

$$\eta = \frac{1}{2\pi} \frac{\int\limits^\pi |\tilde{a}(u)|^2 \{1 - |\tilde{s}(u)|^2\} du}{\sum\limits_{p=p_1, \; p_2, \ldots, \; p_\alpha} |a_p|^2} , \tag{2}$$

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where $\widetilde{a}(u)$ and $\widetilde{s}(u)$ are the complex Fourier series compiled, repectively, from amplitudes of incident waves (a_n) and from elements of the central line of the matrix of scattering (s_n) of the system (see [1]).

With the excitation of only one radiator and an identical load of feeders of the passive elements, characterized by the reflection factor Γ, on the basis of [1] and formula (2), we obtain

$$\eta_p^{(m)} = 2\pi \frac{\int_{-\pi}^{\pi} \frac{1-|\widetilde{s}(u)|^2}{|1-\Gamma\widetilde{s}(u)|^2}\,du}{\left|\int_{-\pi}^{\pi} \frac{du}{1-\Gamma\widetilde{s}(u)}\right|^2} = \eta_p^{(m)} \quad (p = 0, \pm 1, \ldots). \tag{3}$$

With the wave loading of feeders of the parasitic elements ($\Gamma = 0$) from (3) we get the relation

$$\eta_p^{(c)} = \eta_{p_0}^{(c)} = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi}|\widetilde{s}(u)|^2\,du = 1 - \sum_{n=-\infty}^{\infty}|s_n|^2 \quad (p = 0, \pm 1, \ldots). \tag{4}$$

similar to that obtained in [2] for two-dimensional periodic arrays.

Let us consider the case when the radiators with numbers within $-N \leqslant n \leqslant N$ are excited with the linear distribution of the phases and feeders of the remaining loadings of their wave impedances, i.e.,

$$a_n = \begin{cases} |a_n|e^{-in\Phi}, & n = 0, \pm 1, \pm 2, \ldots, \pm N; \\ 0, & n = \pm(N+1), \pm(N+2), \ldots, \pm\infty, \end{cases} \tag{5}$$

where Φ is the difference in the phases between the adjacent radiators.

When $N \gg 1$, and at a sufficiently smooth distribution of the amplitudes $|a_n|$ (i.e., for the highly directional arrays), the periodic function $|\widetilde{a}(u)|^2$ is changed considerably faster than $|\widetilde{s}(u)|^2$ and has in the interval $(-\pi, \pi)$ a single main maximum (when $u = \Phi$). Considering this fact and taking into account the theorem of Parseval, on the basis of (2) we have[1])

$$\eta(\Phi) \simeq 1 - |\widetilde{s}(\Phi)|^2. \tag{6}$$

---

[1]In the limit, when $N \to \infty$, relation (6) becomes precise.

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The radiation power ($W_{изл}$) is a function of characteristics of both the antenna and sources which are the connected system, and, therefore, it is convenient to characterize the change in $W_{изл}$ not by the efficiency, which is a characteristic only of the antenna, but by the so-called output coefficient of power

$$\xi = \frac{W_{изл}}{W_{макс}}, \tag{7}$$

where $W_{макс}$ is the maximally possible total power which can be removed from sources which excite the system.

If the sources are generators of voltage with an emf $\mathscr{E}_n$ and identical internal impedances characterized by the reflection factor Γ, then on the basis of (7), by applying the Parseval identity, we get[2]:

$$\xi = (1 - |\Gamma|^2) \frac{\int_{-\pi}^{\pi} \frac{1 - |\widetilde{s}(u)|^2}{|1 - \Gamma \widetilde{s}(u)|^2} |\widetilde{\mathscr{E}}(u)|^2 du}{\int_{-\pi}^{\pi} |\widetilde{\mathscr{E}}(u)|^2 du}, \tag{8}$$

where $\widetilde{\mathscr{E}}(u)$ is the complex Fourier series compiled of the emf [1].

If the array is excited by (2N+1) voltage generators with linearly phased-in emf [similar to (5)], then under the same assumptions as with the derivation of (6), we obtain

$$\xi(\Phi) \simeq (1 - |\Gamma|^2) \frac{1 - |\widetilde{s}(\Phi)|^2}{|1 - \Gamma \widetilde{s}(\Phi)|^2}. \tag{9}$$

When $\Gamma = 0$ the output coefficient concurs with the efficiency which is natural, since due to the absence of reflection from the generators, the amplitudes of the extraneous waves do not depend on the loads.

For the plane two-dimensional periodic array with the use of notations of work [1], relations (3), (4), (6), and (9) are replaced by the following:

------

[2] Let us recall that the source of voltage generates the maximally possible power on the load with the impedance complex conjugate to the internal impedance of the source.

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$$\eta_{N}^{(H)} - \eta_{N}^{(c)} = 4\pi^2 \frac{\int\limits_{-1}^{1}\int\limits_{-1}^{1} \frac{1 - |\tilde{s}(u, v)|^2}{|1 - \Gamma\tilde{s}(u, v)|^2}\, du\, dv}{\left|\int\limits_{-1}^{1}\int\limits_{-1}^{1} \frac{du\, dv}{1 - \Gamma\tilde{s}(u, v)}\right|^2}. \tag{10}$$

$$\eta^{(c)} \simeq \eta^{(H)} = 1 - \frac{1}{4\pi^2}\int\limits_{-1}^{1}\int\limits_{-1}^{1} |\tilde{s}(u, v)|^2\, du\, dv = 1 - \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} |s_{nm}|^2. \tag{11}$$

$$\eta_i(\Phi, \Psi) \simeq 1 - |\tilde{s}(\Phi, \Psi)|^2. \tag{12}$$

$$\xi(\Phi, \Psi) \simeq (1 - |\Gamma|^2) \frac{1 - |\tilde{s}(\Phi, \Psi)|^2}{|1 - \Gamma\tilde{s}(\Phi, \Psi)|^2}. \tag{13}$$

Let us turn to the linear or two-dimensional periodic array with one active radiator and with the wave loads in feeders of the passive elements. By equating the expressions for the radiated power, computed along the fields in the far zone and in feeders of the system, and using work [1], it is possible to obtain the equality

$$\eta^{(c)} = \int\limits_{\vartheta=0}^{\pi}\int\limits_{\varphi=0}^{2\pi} \left|\vec{f}^{(c)}(\vartheta, \varphi)\right|^2 \sin\vartheta\, d\vartheta\, d\varphi, \tag{14}$$

where $\vec{f}^{(c)}(\vartheta, \varphi)$ and $\eta^{(c)}$ are the partial pattern and directive gain of the element correspondingly for the linear and two-dimensional arrays.

Designating by $D^{(c)}(\vartheta, \varphi)$ the direct gain of the element of the array (with the wave load of the passive feeders) and on the basis of the determination of the directive gain, we get

$$D^{(c)}(\vartheta, \varphi)\eta^{(c)} = 4\pi\left|\vec{f}^{(c)}(\vartheta, \varphi)\right|^2 = g^{(c)}(\vartheta, \varphi), \tag{15}$$

where $g^{(c)}(\vartheta, \varphi)$ is the amplification of the element of the linear or two-dimensional array (with the wave load of the feeders).

Relations (14) and (15) can be examined as conditions of the normalization of the partial patterns.

In conclusion of the section, let us note that the connection between the efficiency, directive gain, gain, and partial pattern of the element in the array with arbitrary but equal loads in the feeders retains the form of the equalities (14) and (15), in which, thus, it is sufficient to replace the superscript (c) with (H) [1].

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Connection between the function of scattering and partial gain of the element in the periodic array

The power radiated by the antenna system can be computed doubly: according to amplitudes of waves in feeders and by fields in the far zone.

Since the amplitudes of the waves are connected by the matrix of scattering, and the fields of radiation are expressed by the superposition of the partial patterns, it appears possible to establish the connection between these most important characteristics of the system. Obtained in works [3, 4] are relationships between amplitudes of waves in the feeders and the power being radiated for finite systems of radiators. By extending these relationships to the linear periodic systems of radiators and considering their characteristics [1]:

$$S_{nm} = s_{|n-m|} = s_p \ (n, \ m, \ p \ 0, \ \pm 1, \ \pm 2, \ \ldots).$$
$$\vec{f}_p^{(c)}(\vartheta, \ \varphi) = \vec{f}_0^{(c)}(\vartheta, \ \varphi) \exp{(i \, pkd \cos{\vartheta})}.$$

it is possible to obtain the equalities

$$Q_q = \sum_{p=-\infty}^{\infty} s_p s_{p-q}^* \ (q = 0, \ \pm 1, \ \pm 2, \ldots);$$ (16)

$$\tau_q = \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} |\vec{f}_0^{(c)}(\vartheta, \ \varphi)|^2 \exp{(- i \, qkd \cos{\vartheta})} \sin{\vartheta} \, d\vartheta \, d\varphi.$$ (17)

in which $q = m' - m$, $Q_q \equiv Q_{mm'}$, $\tau_q \equiv \tau_{mm'}$. i.e., for the array being considered, the elements of the infinite matrices [Q] and [τ] depend actually on the difference in the subscripts. For the two-dimensional periodic arrays, instead of (16) and (17), we have

$$Q_{\alpha\beta} = \sum_{r=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} s_{rl} s_{r-\alpha, l-\beta}^* \ (\alpha, \ \beta = 0, \ \pm 1, \ \pm 2 \ldots);$$ (18)

$$\tau_{\alpha\beta} = \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} |\vec{f}_{00}^{(c)}(\vartheta, \ \beta)|^2 \exp{[- i \, \alpha kd_1 \sin{\vartheta} \cos{\varphi} -}$$
$$- i \beta kd_2 \sin{\vartheta} \sin{\varphi}] \sin{\vartheta} \, d\vartheta \, d\varphi$$
$$(\alpha \ \beta = 0, \ \pm 1, \ \pm 2, \ldots).$$ (19)

where α and β are differences in the subscripts, respectively, with respect to the rows and columns.[1]

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[1] Let us note that in the derivation of relations (17) and (19), it was assumed that in the infinite arrays there exists no continuous

18   [continued on next page]

Quantities τ_q for the linear and $\tau_{\alpha\beta}$ for the two-dimensional arrays are coefficients of the interaction of partial patterns of definite radiators of the considered systems; they are distinguished from similar coefficients introduced in works [3] and [4] only by the factor caused by the normalization of the patterns. In conformity with (14), (17) and (19), the quantities τ_0 and τ_{00} have physical meaning of the efficiency of radiators of corresponding arrays (with wave loading of feeders of the remaining elements); τ_q (when $q \neq 0$) and $\tau_{\alpha\beta}$ (when $\alpha^2 + \beta^2 \neq 0$) are measures of the nonorthogonality of the partial patterns.

It is simplest of all to explain the meaning of elements of the infinite matrices [Q] for the arrays being considered on the basis of relations (22) and (27) (see below), from which, in particular, there follow almost evident equalities [2, 3, 4]:

$$Q_0 \sum_{n=-\infty}^{\infty} |s_n|^2 = 1 - \tau_0^{(r)},$$

$$Q_{00} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} |s_{np}|^2 = 1 - \tau_{00}^{(r)}.$$

By turning at first to the linear array, let us introduce the notations:

$$u = kd \cos\vartheta, \tag{20a}$$

$$g(u) \quad \frac{1}{2\pi} \int_0^{2\pi} g_0^{(r)} \left(\arccos\frac{u}{kd}, \varphi \right) d\varphi. \tag{20b}$$

where $g(u)$ is the partial gain of the element of the array (as a function of the generalized coordinate u) averaged over the angle φ.

By taking (20a and b) and (15) into account instead of (17), we obtain the equality equivalent to it

$$\tau_q \quad \frac{1}{2kd} \int_{-kd}^{kd} g(u') e^{-iqu'} du'. \tag{21}$$

By using works [2, 3], in notations (16) and (17) it is possible to obtain an infinite set of equalities

$$\delta_{q0} \quad Q_q \quad \tau_q (q \quad 0, \; \pm 1, \; \pm 2, \ldots). \tag{22}$$

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[continued from previous page] surface waves; in the opposite case, it would be necessary to consider them in the energy balance of the system.

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By multiplying each of the relations of (22) by e^{iqu} and adding them with respect to q in limits of $-\infty$, $+\infty$, we get

$$1 - \widetilde{Q}(u) \quad \widetilde{\tau}(u).$$ (23a)

where

$$\widetilde{Q}(u) \quad \sum_{q=-\infty}^{\infty} Q_q e^{iqu} ; \quad \widetilde{\tau}(u) = \sum_{q=-\infty}^{\infty} \tau_q e^{iqu}.$$ (23b

On the basis of (16), (21) and 23b) we get [1]

$$\widetilde{Q}(u) \quad |\widetilde{s}(u)|^2.$$ (24)

$$\widetilde{\tau}(u) \quad \frac{\pi}{kd} \int_{-kd}^{kd} g(u') \delta_{2\pi}(u - u') du'$$

$$= \frac{\pi}{kd} \sum_{n=-\infty}^{\infty} g(u - 2\pi n) \sigma \left(\frac{u - 2\pi n}{kd} \right).$$ (25a)

where

$$\delta_{2\pi}(\xi) \quad \frac{1}{2\pi} \lim_{N \to \infty} \sum_{q=-N}^{N} e^{iq\xi}$$ (25b)

is the periodic delta function [5]:

$$\sigma(z) \quad \begin{cases} 0, & |z| > 1, \\ 1/2, & |z| - 1, \\ 1, & |z| < 1. \end{cases}$$ (25c)

By substituting (24) and (25a) into (23a) and returning to the angular coefficients, we obtain

$$1 - |\widetilde{s}(kd \cos \vartheta)|^2 \quad \frac{\lambda}{2d} \sum_{n} \left\{ \frac{1}{2\pi} \int_{0}^{2\pi} g_0^{(c)}(\vartheta_n, \psi) d\vartheta \sigma(\cos \vartheta_n) \right\}.$$ (26a)

where the summation (at fixed ϑ) is conducted according to discrete angles determined by the euality

$$\cos \vartheta_n \quad \cos \vartheta - n \frac{\lambda}{d}.$$ (26b)

where the summation index is actually included in the limits of

$$- E\left[\frac{d}{\lambda}(1 - \cos \vartheta) \right] < n < E\left[\frac{d}{\lambda}(1 + \cos \vartheta) \right].$$ (26c)

Function E(z) denotes the whole part of the number z.

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[1] In the process of the conversions of (16) to the form of (24), the order of summation (with replacement: r = q-p, q = p+r); with the transition from (21) to (25a) the order of the summation and integration is changed.

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For the two-dimensional periodic arrays, instead of (22) and (23a), we obtain

$$\delta_{\alpha\mu}\delta_{\beta\mu} \quad Q_{\alpha\mu} \quad \tilde{\tau}_{\alpha\beta}(\alpha,\beta \quad 0, \pm 1, \pm 2, \ldots); \qquad (27)$$

$$1 \quad \tilde{Q}(u, v) \quad \tilde{\tau}(u, v). \qquad (28)$$

where $\tilde{Q}(u, v), \tilde{\tau}(u, v)$ are the appropriate two-dimensional Fourier series.

By transforming (18) similar to the linear case, we obtain

$$\tilde{Q}(u, v) = |\tilde{s}(u, v)|^2. \qquad (29)$$

To obtain the function $\tilde{\tau}(u, v)$, let us transform, at first, the expression (19), introducing the generalized coordinates:

$$u' \quad kd_1\sin\vartheta\cos\varphi, \quad v' = kd_2\sin\vartheta\sin\varphi. \qquad (30a)$$

The element of the solid angle, according to (30a), equals

$$\sin\vartheta\, d\vartheta\, d\varphi \quad \frac{D(\vartheta,\varphi)}{D(u',v')} \quad \frac{du'dv'}{kd_1kd_2\sqrt{1-\left(\frac{u'}{kd_1}\right)-\left(\frac{v'}{kd_2}\right)^2}} \qquad (30b)$$

In connection with the two-valued property of function $\vartheta(u', v')$, let us divide the integral of (19) into the sum of integrals

$$\tau_{\alpha\beta} = \tau_{\alpha\beta}^{(+)} + \tau_{\alpha\beta}^{(-)},$$

taken, respectively, over the upper (+) and lower (−) hemispheres, which are regions of single-valuedness for $\vartheta(u', v')$. By introducing into $\tau_{\alpha\beta}^{(-)}$ the replacement $\vartheta \to \pi - \vartheta'$ and considering that $\sin\vartheta' = \sin\vartheta$, for $\tau_{\alpha\beta}$ it is possible to obtain the following expression:

$$\tau_{\alpha\beta} \quad \frac{1}{4\pi\, kd_1kd_2} \iint\limits_{(C_R)} |g^{(+)}(u', v') + g^{(-)}(u', v')| \times$$
$$\times \exp(-i\alpha u' - i\beta v') \frac{du'dv'}{\cos\vartheta(u_1, v')}. \qquad (31a)$$

where

$$g^{(+)}(u', v') = g_{00}^{(c)}|\vartheta(u', v'), \varphi(u', v')|,$$
$$g^{(-)}(u', v') = g_{00}^{(c)}[\pi - \vartheta(u', v'), \varphi(u', v')];$$
$$0 \leq \vartheta(u', v') = \text{Arc}\sin\sqrt{\left(\frac{u'}{kd_1}\right)^2+\left(\frac{v'}{kd_2}\right)^2} < \frac{\pi}{2}. \qquad (31b)$$

$$0 \leq \varphi(u', v') = \text{Arc tg}\frac{v'd_1}{u'd_2} \quad 2\pi. \qquad (31c)$$

The integration in (31a) is extended to the area of the "ellipse of radiation"

$$\left(\frac{u'}{kd_1}\right)^2+\left(\frac{v'}{kd_2}\right)^2 \leq 1. \qquad (31d)$$

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which is a representation of the upper half-space on the plane (u', **v'**).

By transforming (31a) similar to the linear case, we get

$$\widetilde{\tau}(u, v) \simeq \frac{\lambda^2}{4\pi} \iint_{(^{+}_R)} \left[\frac{g(u', v')}{\cos \vartheta(u'v')} - \delta_2(u - u')\delta_2(v - v') \right]$$

$$\times du'dv' \simeq \frac{\lambda^2}{4\pi A} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} \frac{g(u - 2\pi n, v - 2\pi p)}{\cos \vartheta(u - 2\pi n, v - 2\pi p)}$$

$$\times \sigma \left[\sqrt{\left(\frac{u - 2\pi n}{kd_1} \right)^2 + \left(\frac{v - 2\pi p}{kd_2} \right)^2} \right]. \tag{32a}$$

where

$$g(u, v) = g^{(+)}(u, v) + g^{(-)}(u, v), \tag{32b}$$

$A = d_1 \times d_2$ is the area of aperture of the array per one element; $\sigma[...]$ is determined by (25c).

On the basis of (24), (31b) and (32b), it is possible to write equality (23a) in the following coordinate form:

$$1 - |\widetilde{s}(kd_1 \sin \vartheta \cos \varphi, kd_2 \sin \vartheta \sin \varphi)|^2$$

$$= \frac{\lambda^2}{4\pi A} \sum_n \sum_p \frac{g_{\text{sn}}^{(c)}(\vartheta_{np}, \varphi_{np}) + g_{\text{sn}}^{(c)}(\pi - \vartheta_{np}, \varphi_{np})}{\cos \vartheta_{np}} \sigma(\sin \vartheta_{np}). \tag{33a}$$

The summation (at fixed ϑ, φ) is conducted along discrete directions $(\vartheta_{np}, \varphi_{np})$, determined by relations

$$\sin \vartheta_{np} \cos \varphi_{np} = \sin \vartheta \cos \varphi - n \frac{\lambda}{d_1};$$

$$\sin \vartheta_{np} \sin \varphi_{np} = \sin \vartheta \sin \varphi - p \frac{\lambda}{d_2}. \tag{33b}$$

where

$$0 \leqslant \vartheta_{np} \leqslant \pi/2, \ 0 \leqslant \varphi_{np} < 2\pi,$$
$$\vartheta = \vartheta_{00}, \ \varphi = \varphi_{00}. \tag{33c}$$

When $d_1 = d_2$ and $g_{00}^{(-)}(\vartheta, \varphi) = 0$ relation (33a) concurs with that obtained in work [2].

In conclusion to the section, let us note that in relations (26a) and (33a) the slipping directions[1] should be excluded from the examination, since for the finite arrays in these directions the concept

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[1]That is, the directions corresponding to $\vartheta = 0$ and $\vartheta = \pi$ for the linear arrays and $\vartheta = \pi/2$ for the two-dimensional arrays.

of partial radiation patterns is incorrect, and it is necessary to use the p⁻ ial fields. Therefore, in formula (25c) the value of $\sigma(1) = 1$, .as a purely formal meaning conditioned by the apparatus of the Fourier series.

Certain theoretical limitations of characteristics of linear arrays

Relations (26a) and (33a) are equivalent to the law of the conservation of energy for periodic systems of radiators and establish the close connection between the matrix of scattering of the system and radiation patters of the radiators. Hence, in particular, it follows that in the analysis of the multi-element arrays it is impossible to assign the patterns of the radiators irrespective of the matrix of scattering, especially, if the period (or periods) of the array does not exceed the wavelength.

The indicated relations make it possible to expand a number of interesting regularities peculiar to the periodic radiating systems. For the two-dimensional arrays (when $d_1 = d_2$) these regularities were first studied in work [2]; below some of them are examined in reference to the linear periodic arrays.

Since the right side of (26a) is the sum of the negative functions, resulting from this relation are three important inequalities[1]:

$$0 \leqslant |\widetilde{s}(kd\cos\vartheta)| \leqslant 1; \tag{34}$$

$$0 \leqslant \sum_n g_{\text{cp}}^{(c)}(\vartheta_n)\,\sigma(\cos\vartheta_n) \leqslant \frac{2d}{\lambda}; \tag{35a}$$

$$0 \leqslant g_{\text{cp}}^{(c)}(\vartheta) \leqslant \frac{2l}{\lambda}, \tag{35b}$$

where

$$g_{\text{cp}}^{(c)}(\vartheta) = \frac{1}{2\pi}\int_0^{2\pi} g_0^{(c)}(\vartheta, \varphi)\,d\varphi \tag{35c}$$

is the partial gain averaged over φ.

The upper limit of the gain, equal, according to (35b), to $\frac{2d}{\lambda}$, becomes physically obvious if we consider that for the multi-element axisymmetric phased array with equal-amplitude excitation of the

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[1] Let us note that the relation (34) has already been obtained (true, strictly insufficiently) in work [1].

. . . ., the gain in the maximum does not exceed $\frac{2Nd}{\lambda}$ (N - the number of elements) and is connected with the gain of the radiator by the known relation [2, 6]

$$G_{\text{макс}} = N g^{(c)} (\vartheta_{\text{макс}}). \tag{36}$$

When $d < \frac{\lambda}{2}$ the sum in the right side of (26a) is reduced to a single term, and the scattering function in the interval $-\pi \leqslant u \leqslant \pi$ satifies the relations

$$|\tilde{s}(u)|^2 = \begin{cases} 1 & \text{[when] при } -\pi \leqslant u \leqslant kd, \\ 1 - \frac{\lambda}{2d} g(u) & \text{[when] при } -kd < u - kd\cos\vartheta < kd, \\ 1 & \text{[when] при } kd \leqslant u \leqslant \pi. \end{cases} \tag{37}$$

from which, in particular, it follows that the defined scattering matrix corresponds when $d < \frac{\lambda}{2}$ to the completely defined partial gain averaged over φ (the reverse is incorrect!).

In connection with this, it is interesting to note that due to the eveness of $\tilde{s}(u)$ (see [1]) function $g_{cp}^{(c)}(\vartheta)$, in conformity with (37), proves to be symmetric with respect to the plane perpendicular to the axis of the array. In this case the pattern of the radiator, taken isolated from the system, can be nonsymmetric.

From (37) it also follows that the efficiency of the linear phased array [see (4)] with the period within the half-wave is equal to zero when $kd < |\Phi| < \pi$; this phenomenon is physically evident, since with such phasing the considered array does not have the main beams in the region of the real angles[1].

On the basis of formulas (4), (35b) and (37), it is possible to obtain the following inequality:

$$\eta_0^{(c)} \leqslant \frac{2d}{\lambda} < 1. \tag{38}$$

From (38) it follows that, in the first place, the efficiency of the radiator of the considered array (with the wave load of feeders

[1] Such a behavior of the efficiency is the result of the infinity of the array and the absence of Joule losses in the radiators and the surrounding space. For the phased array of finite dimensions, in the case of the disappearance of the beam, in practice only its extreme elements radiate, and the efficiecy proves to be of the order of 1/N.

of the remaining elements) does not exceed $\frac{2d}{\lambda}$ and, in the second place, even if one element of the scattering matrix of the system is different from zero.

A somewhat more specific judgment about the elements of the matrix [s] can be obtained on the basis of (37).

For example, it is possible to show that the radiators can be decoupled $(s_{\pm 1} = s_{\pm 2} = \ldots = 0)$ only in the trivial case when $|s_0| = 1$, i.e., when they are not radiating. Actually, formulas (37) are identically satisfied when $|\tilde{s}(u)| \equiv |s_0| = 1$ and $g_0^{(c)}(\vartheta, \varphi) = 0$, and when $|s_0| < 1$ and $s_n = 0 (n \neq 0)$, on the basis of (4), (15) and (37), we get the inequality

$$D_{c\varphi}^{(c)}(\vartheta) = \frac{2d}{\lambda} \frac{1 - |\tilde{s}(kd \cos \vartheta)|^2}{\eta^{(c)}} = \frac{2d}{\lambda} < 1, \tag{39}$$

which, obviously, contradicts the definition of efficiency.

Let us note that the inequality (38) doe not at all prohibit the reaching of a 100% efficiency with operation of the array in the scanning mode (i.e., with the linear-phase excitation of its elements). According to (6) and (34), for this the vanishing of function s(u) is necessary when $-kd < u < kd$, i.e., in the interval of the real angles. From (37) it follows that the scattering function of the indicated "ideal" phased array satisfies the conditions:

$$|\tilde{s}_{ид}(u)| = \begin{cases} 0 \text{ [when]}_{при} -kd < u < kd, \\ 1 \text{ when}_{при} kd < |u| < \pi, \end{cases} \tag{40}$$

and the partial gain

$$g_{ид}^{(c)}(u) = \frac{2d}{\lambda} = \text{const}. \tag{41}$$

i.e., its radiators are omnidirectional[1]. The efficiency of the element of the ideal array, as follows from (41), reaches its upper limit equal to $\frac{2d}{\lambda}$. Elements of the matrix $[s_{ид}]$, according to (40), are expressed in the form

$$s_{ид\,n} = \frac{1}{\pi} \int_{kd}^{\pi} e^{j\psi\,(u)} \cos nu\,du \tag{42}$$

$$(n = 0, \pm 1, \pm 2, \ldots).$$

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[1] The omnidirectionality, which, as is known, is impossible at finite distributions of current, here is the result of the infinity of the considered systems in the absence of Joule losses in radiators and the surrounding space.

where $\psi(u)$ is a certain real function.

In connection with the expressed remark, it is clear that the concordance of the phased array, for which $|\tilde{s}(kd\cos\vartheta_{...})|$ 1. is necessary, and the concordance of its elements in the generally accepted meaning ($|s_0| \ll 1$) are completely different concepts. In a number of cases, for example, with the closely located ($d \ll \lambda$) elements, the concordance of the radiators can lead even to a mismatch in the phased array, and, on the other hand, the wide-angled matching of the latter can correspond noticeably to the detuned elements.

Regarding the arrays with a period of $d > \dfrac{\lambda}{2}$, then for them many of the limitations noted above drop out. In particular, it becomes fundamentally permissible that $s_0 = s_{+1} = s_{+2} = \ldots = 0$ and $\eta^{(r)} = \eta(\Phi) = 1$. The mutually single-valued correspondence of type (37) between the gain of the radiator and the modulus of the scattering function of the system is preserved when $d < \lambda$ within $-(2\pi - kd) < u < 2\pi - kd$, which correspond only to part of the spectrum of the real angles [$\pm$arc sin X $(\dfrac{\lambda}{d}-1)$ from the normal to the aperture][1].

Conclusions

1. Expressions are obtained for the efficiency and power output coefficient for linear and two-dimensional periodic arrays with excitation by their voltage generators [oscillators] with identical internal impedances and arbitrary emfs.

2. The relationship of the connection between the scattering matrix and the radiator gain in the linear periodic array has been established. For the two-dimensional array, this relationship is obtained under more general assumptions than it is in [2], and, namely, when the periods of the array are arbitrary, and the radiation of energy is not limited by the half-space.

3. On the basis of the relationship of the connection between the scattering matrix and the radiator gain of the linear periodic

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[1] However, as follows from (26a), the defined function $g_0^{(c)}(\vartheta, \varphi)$ at any d corresponds only to one function $|\tilde{s}(u)|$. The reverse correspondence (between $g_0^{(c)}(\vartheta)$ and $s(u)$ is correct only when $d < \dfrac{\lambda}{2}$.

array, a number of theoretical limitations for its characteristics is obtained, in particular:

a) the gain of the radiator, averaged over φ , of the linear array (with a wave load of feeders of all elements) does not exceed $2d/\lambda$;

b) when $d < \lambda$, the gain indicated in item a) is a function symmetric with respect to the plane perpendicular to the axis within limits of $\pm \arcsin (\lambda/d-1)$;

c) the interconnection between radiators of the array with the period $d < \lambda/2$ is inevitable;

d) the efficiency of the radiator (with the wave load of feeders of the remaining elements) when $d < \lambda/2$ does not exceed $2d/\lambda$.

e) with a period exceeding $\lambda/2$, the radiators of the array in principle can be decoupled and matched, and their efficiency can reach 100%.

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